ρ and σ Mesons in Unitarized Thermal χPT

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We present our recent results for the ρ and σ mesons considered as resonances in pion-pion scattering in a thermal bath. We use chiral perturbation theory to order p^4 for the low energy behaviour, then extend the analysis via the unitarization method of the Inverse Amplitude into the resonance region. The width of the rho broadens about twice the amount required by phase space considerations alone, its mass staying practically constant up to temperatures of order 150 MeV. The sigma meson behaves in accordance to chiral symmetry restoration expectations.

Triggered by the possibility of exploiting the dilepton spectrum as a signal of the Quark and Gluon Plasma in Relativistic Heavy Ion Collisions, there have been numerous studies of the thermal behaviour of the ρ resonance in a hot hadron medium. Some of these theoretical approaches are summarized in figure (1). The wide spread of predictions signals a strong model dependence in the various calculations, although a general (not universal) trend points to a natural broadening of the width due to a larger available phase space. In this work we apply chiral perturbation theory [10](χPT) complemented with the Inverse Amplitude Method [11] (IAM) to obtain predictions guided only by the principles of chiral symmetry and unitarity. These combined methods provide outstanding fits to low energy pion scattering data [11] thus fixing the low energy constants of the chiral lagrangian. We use this same constants in our thermal amplitudes and all finite temperature results fall as predictions.

The starting point is a calculation of the thermal pion scattering amplitude to one loop in chiral perturbation theory, which includes calculating pion loops and tadpoles (where temperature dependence is included through Matsubara sums in the imaginary time formalism) and polynomial counterterms with arbitrary coefficients (the low energy constants) which are fitted to known pion scattering phase shifts. Since the thermal bath induces a preferred reference frame, our results [12] can be best written in terms of the $\pi_+ \ \pi_- \longrightarrow \pi_0 \ \pi_0$ scattering amplitude $A(\mathbf{S},\mathbf{T},\mathbf{U},\beta)$ which depends on the four vectors $\mathbf{S} = p_{\pi_+} + p_{\pi_-} ...$ generalizing the Mandelstam variables and allowing us to generate the other possible pion scattering amplitudes by crossing symmetry.

At finite temperature and for $s > 4m_\pi^2$ and below other thresholds, the partial wave projections obtained from our amplitude satisfy the unitarity relation perturbatively, that is

$$\text{Im } a_2(s) = 0 \ , \quad \text{Im } a_4(s;\beta) = \sigma_\beta(\mathbf{S}_0)|a_2(s)|^2 \ , \ \mathbf{S}_0 > 2m_\pi \ , \tag{1}$$

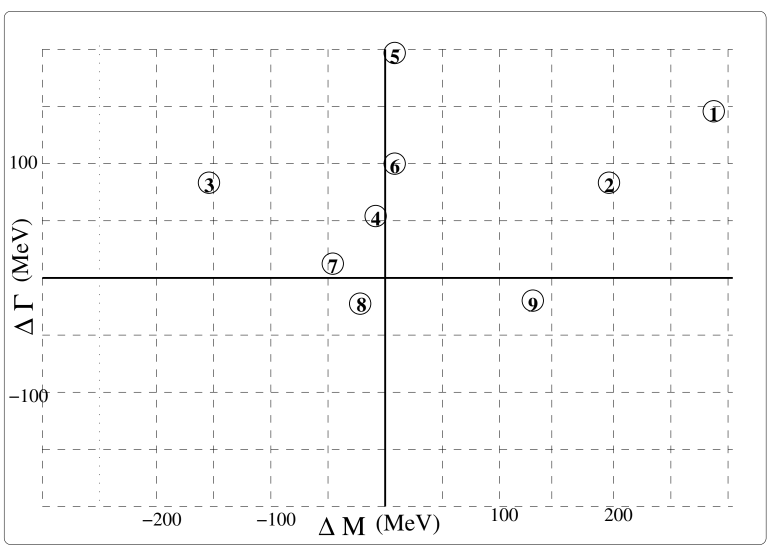

FIG. 1: Various existing calculations on the thermal (at about T=150 MeV) vector meson mass and width. The numbers refer to the bibliography.

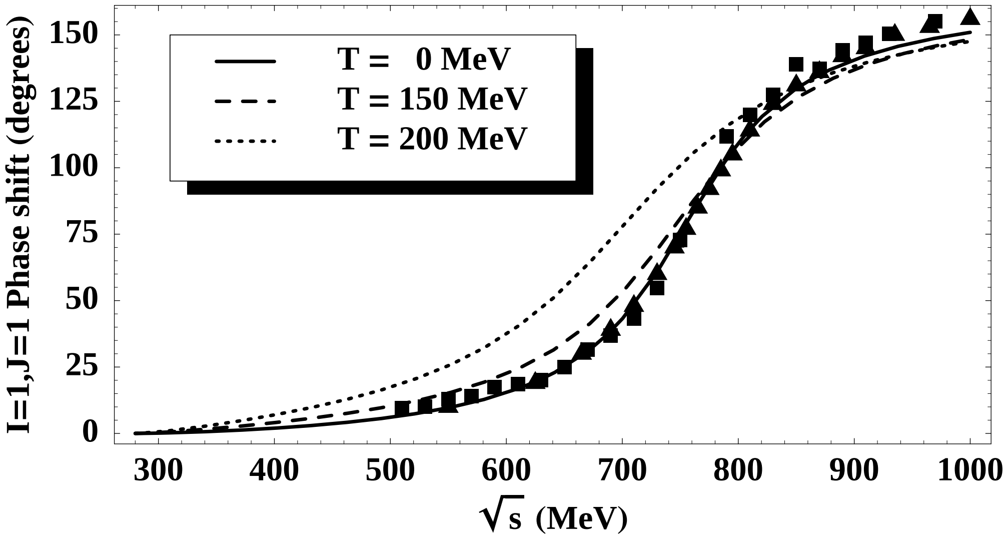

FIG. 2: Zero and finite temperature pion scattering phase shifts in the ρ channel.

with a thermal phase space

$$\sigma_\beta(E) = \sqrt{1 - \frac{4m_\pi^2}{E^2}} \left[1 + \frac{2}{\exp(\beta|E|/2) - 1} \right] \ . \tag{2}$$

The temperature is of order p, M_π in the chiral counting, and thus for small temperatures and momenta all corrections scale at least as the fourth power of the small quantity $M_\pi/(4\pi f_\pi)$, providing very good accuracy in this regime. Alas, the chiral expansion is intrinsically a polynomial with chiral logarithms (from pion loops) providing the imaginary parts in (1) only perturbatively, but violating severely exact unitarity already around the ρ resonance region. Still, from the behaviour of the phase shifts at low energy, we find a model independent thermal enhancement of the low energy interactions, conserving their attractive or repulsive nature. This effect is consistent with a thermal increase of the rho width and an almost constant mass[12]. To extend this approach to momenta and temperatures beyond the reach of one loop χPT, several unitarization methods have been proposed. Here we employ the well tested IAM (see [13] for a thorough introduction). In terms of the χPT momentum expansion, one approximates $a(s)$ by

$$a^{\rm IAM}(s;\beta) = \frac{a_2^2(s)}{a_2(s) - a_4(s;\beta)} \tag{3}$$

satisfying

$$\text{Im } a^{\rm IAM} = \sigma_\beta(s) |a^{\rm IAM}(s;\beta)|^2 \ . \tag{4}$$

One can think of a particular unitarization method giving an exact treatment of the imaginary part of the inverse amplitude but just an approximation to the real part; the IAM uses the $O(p^4)$ χPT amplitude for the approximation and can be considered as the [1,1] Padé approximant for the χPT series in inverse powers of $(4\pi f_\pi)^2$. Further, the rational formula (3) allows the treatment of one pole resonance in each IJ channel which can be tracked down in the second Riemann sheet for complex s. If we think of the doubly subtracted dispersion relation satisfied by $a(s)$, the left cut corresponding to t-channel pion loops is only approximately considered, whereas the right s-channel cut is exactly taken into account. This induces a small crossing violation, but its impact on the resonance region is numerically controlled because of the large distance of this cut to the ρ or σ poles in the complex plane.

Figure (2) displays the IAM ρ channel phase shift in pion scattering, at zero temperature (compared to experimental data) and the finite temperature prediction. The results are consistent with the evolution of the σ and ρ poles in the complex plane shown in fig.(3) (the analytical extension of the loop and tadpole integrals to the complex plane is described in [14]).

The behaviour of the σ hints at chiral symmetry restoration: observe how its mass decreases as the temperature raises and how its width, initially increasing, dramatically drops when approaching the relatively stable two pion threshold. On the other hand, the width of the ρ increases in all the considered temperature range. This calculation,

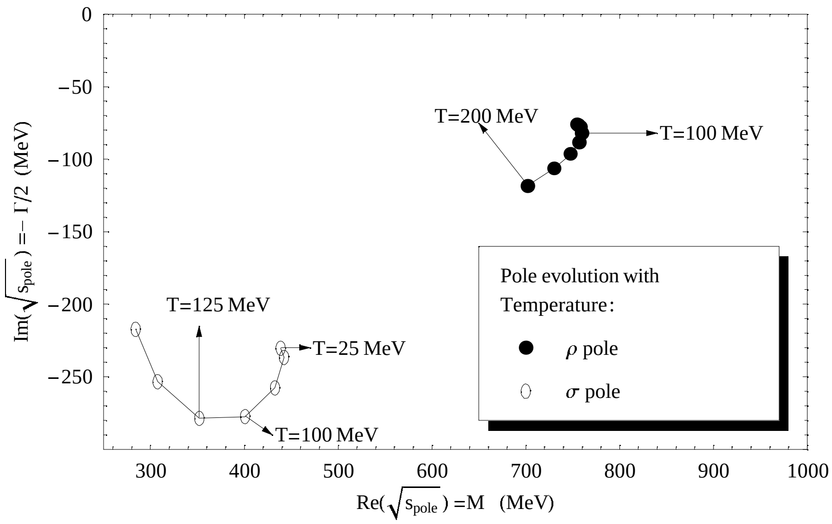

FIG. 3: How dinamically generated resonance parameters evolve with the temperature.

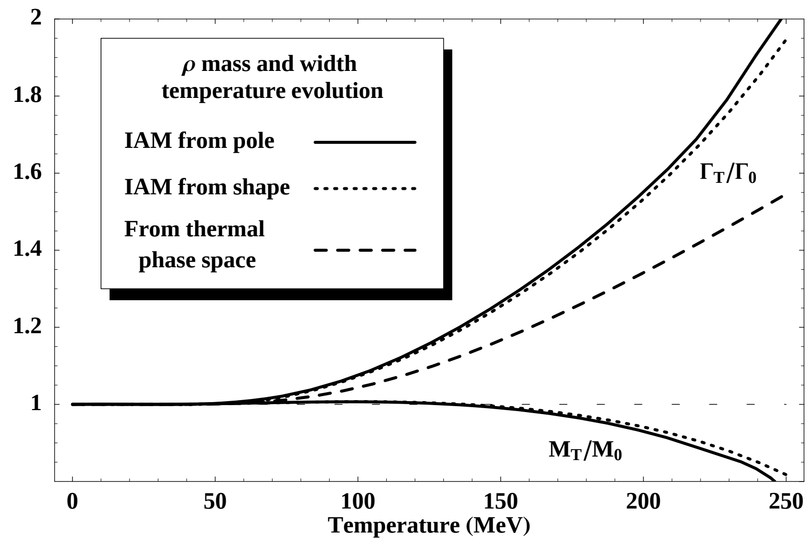

FIG. 4: Thermal evolution of the ρ mass, width and $\rho\pi\pi$ effective vertex.

performed ab initio in the chirally broken phase, does not have direct access to the phase transition, but serves to pin down models which do have this capability providing them with a general behaviour at low temperatures.

Since in our approach resonances are not explicitly included, but arise dynamically, we can predict the temperature evolution of their masses and couplings. If we specifically concentrate on ρ meson properties, replotted for convenience in figure 4, we find that the thermal width grows faster than the naive expectation from the well known [4] space phase in eqn.(2), the effective $g_{\rho\pi\pi}$ acquires a temperature dependence which was ignored in most of previous works, but in agreement at low temperatures with an old calculation[15] and significantly the ρ mass, which stays practically constant, in agreement with [7] and specially [4] (both papers offering an explanation of the dilepton excess below M_ρ).

Acknowledgements Work supported by grants FPA2000-0956, PB98-0782 and BFM2000-1326. J. R. P. acknowledges also support from the CICYT-INFN collaboration grant 003P 640.15.

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